1- [Griffiths 9.19] (a) Suppose you imbedded some free charge in a piece of glass. About how long would it take for the charge to flow to the surface? (b) Silver is an excellent conductor, but it’s expensive. Suppose you were designing a microwave experiment to operate at a frequency of \(10^{16}\) Hz. How thick would you make the silver coatings? (c) Find the wavelength and propagation speed in copper for radio waves at 1 MHz. Compare the corresponding values in air (or vacuum).

2- [Griffiths 9.20] (a) Show that the skin depth in a poor conductor \((\sigma \ll \omega \varepsilon)\) is \((2/\sigma)\sqrt{\varepsilon/\mu}\) (independent of frequency). Find the skin depth \((\text{in meters})\) for (pure) water. (Use the static values of \(\varepsilon, \mu,\) and \(\sigma\); your answers will be valid, then, only at relatively low frequencies.) (b) Show that the skin depth in a good conductor \((\sigma \gg \omega \varepsilon)\) is \(\lambda/2\pi\) (where \(\lambda\) is the wavelength \(\text{in the conductor}\)). Find the skin depth \((\text{in nanometers})\) for a typical metal \((\sigma \approx 10^7 (\Omega \cdot m)^{-1})\) in the visible range \((\omega = 10^{15}/s)\), assuming \(\varepsilon \approx \varepsilon_0\) and \(\mu \approx \mu_0\). Why are metals opaque? (c) Show that in a good conductor the magnetic field lags the electric field by 45°, and find the ratio of their amplitudes. For a numerical example, use the “typical metal” in part (b).

*3- [Griffiths 9.21] (a) Calculate the (time-averaged) energy density of an electromagnetic plane wave in a conducting medium (Eq. 9.138). Show that the magnetic contribution always dominates. [Answer: \((k^2/2\mu_0^2)E_e^2 e^{-2\kappa z}\)] (b) Show that the intensity is \((k/2\mu_0)E_e^2 e^{-2\kappa z}\).

4- [Griffiths 9.22] Calculate the reflection coefficient for light at an air-to-silver interface \((\mu_1 = \mu_2 = \mu_0, \varepsilon_1 = \varepsilon_0, \sigma = 6 \times 10^7 (\Omega \cdot m)^{-1})\), at optical frequencies \((\omega = 4 \times 10^{15}/s)\).

*5- [Griffiths 9.39] According to Snell’s law, when light passes from an optically dense medium into a less dense one \((n_1 > n_2)\) the propagation vector \(k\) bends away from the normal (Fig. 9.28). In particular, if the light is incident at the critical angle
\[
\theta_c \equiv \sin^{-1}(n_2/n_1),
\]
then \(\theta_T = 90^\circ\), and the transmitted ray just grazes the surface. If \(\theta_I \text{ exceeds } \theta_c\), there is no refracted ray at all, only a reflected one (this is the phenomenon of total internal reflection, on which light pipes and fiber optics are based). But the fields are not zero in medium 2; what we get is a so-called evanescent wave, which is rapidly attenuated and transports no energy into medium 2.

A quick way to construct the evanescent wave is simply to quote the results of Sect. 9.3.3, with \(k_T = \omega n_2/c\) and
\[
k_T = k_T (\sin \theta_T \hat{x} + \cos \theta_T \hat{z});
\]
the only change is that
\[
\sin \theta_T = \frac{n_1}{n_2} \sin \theta_I
\]
is now greater than 1, and
\[
\cos \theta_T = \sqrt{1 - \sin^2 \theta_T} = i \sqrt{\sin^2 \theta_T - 1}
\]
is imaginary. (Obviously, \(\theta_T\) can no longer be interpreted as an angle!)
(a) Show that
\[
\tilde{E}_T(r, t) = E_{0T} e^{-\kappa z} e^{i(kx - cT)},
\]
where
\[
\kappa \equiv \frac{\omega}{c} \sqrt{(n_1 \sin \theta_I)^2 - n_2^2} \quad \text{and} \quad k \equiv \frac{\omega n_1}{c} \sin \theta_I.
\]
This is a wave propagating in the x direction \((\text{parallel to the interface})\), and attenuated in the z direction. (b) Noting that
\[
\alpha \equiv \frac{\cos \theta_T}{\cos \theta_I}
\]
(9.108)
is now imaginary, use

$$\tilde{E}_{0R} = \left( \frac{\alpha - \beta}{\alpha + \beta} \right) \tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left( \frac{2}{\alpha + \beta} \right) \tilde{E}_{0I}$$  \hspace{1cm} (9.109)

to calculate the reflection coefficient for polarization parallel to the plane of incidence. [Notice that you get 100% reflection, which is better than at a conducting surface (see, for example, Prob. 9.22).]

(c) Do the same for polarization perpendicular to the plane of incidence (use the results of Prob. 9.17).

(d) In the case of polarization perpendicular to the plane of incidence, show that the (real) evanescent fields are

$$\begin{align*}
E(r, t) &= E_0 e^{-\kappa z} \cos(kx - \omega t) \hat{y}, \\
B(r, t) &= \frac{E_0}{\omega} e^{-\kappa z} [\kappa \sin(kx - \omega t) \hat{x} + k \cos(kx - \omega t) \hat{z}] .
\end{align*}$$  \hspace{1cm} (7)

(e) Check that the fields in (d) satisfy all of Maxwell’s equations (Eq. 9.67).

(f) For the fields in (d), construct the Poynting vector, and show that, on average, no energy is transmitted in the $z$ direction.