Problem Set 9

due Monday 29th of Azar in TA class

***1-** [Griffiths 5.41] A current I flows to the right through a rectangular bar of conducting material, in the presence of a uniform magnetic field $\mathbf{B}$ pointing out of the page (Fig. 5.56).
(a) If the moving charges are *positive*, in which direction are they deflected by the magnetic field? This deflection results in an accumulation of charge on the upper and lower surfaces of the bar, which in turn produces an electric force to counteract the magnetic one. Equilibrium occurs when the two exactly cancel. (This phenomenon is known as the **Hall effect**.)
(b) Find the resulting potential difference (the **Hall voltage**) between the top and bottom of the bar, in terms of B, v (the speed of the charges), and the relevant dimensions of the bar.
(c) How would your analysis change if the moving charges were *negative*? [The Hall effect is the classic way of determining the sign of the mobile charge carriers in a material.]

2- [Griffiths 5.7] For a configuration of charges and currents confined within a volume $\mathcal{V}$, show that

$$\int_{\mathcal{V}} \mathbf{J} d\tau = d\mathbf{p}/dt, \tag{1}$$

where $\mathbf{p}$ is the total dipole moment. [Hint: evaluate $\int_{\mathcal{V}} \nabla \cdot (x\mathbf{J}) d\tau$.]

3- [Griffiths 5.13] Suppose you have two infinite straight line charges λ, a distance d apart, moving along at a constant speed v (Fig. 5.26). How great would v have to be in order for the magnetic attraction to balance the electrical repulsion? Work out the actual number. Is this a reasonable sort of speed?

4- [Griffiths 5.19] In calculating the current enclosed by an Amperian loop, one must, in general, evaluate an integral of the form

$$I_{\text{enc}} = \int_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a}. \tag{2}$$

The trouble is, there are infinitely many surfaces that share the same boundary line. Which one are we supposed to use?

5- [Griffiths 5.20] **(a)** Find the density ρ of mobile charges in a piece of copper, assuming each atom contributes one free electron. [Look up the necessary physical constants.]
(b) Calculate the average electron velocity in a copper wire 1 mm in diameter, carrying a current of 1 A. [Note: This is literally a *snail's* pace. How, then, can you carry on a long distance telephone conversation?]
(c) What is the force of attraction between two such wires, 1 cm apart?
(d) If you could somehow remove the stationary positive charges, what would the electrical repulsion force be? How many times greater than the magnetic force is it?

6- [Griffiths 5.21] Is Ampère's law consistent with the general rule (Eq. 1.46) that divergence-of-curl is always zero? Show that Ampère's law *cannot* be valid, in general, outside magnetostatics. Is there any such "defect" in the other three Maxwell equations?

***7-** [Griffiths 5.47] The magnetic field on the axis of a circular current loop (Eq. 5.41) is far from uniform (it falls off sharply with increasing z). You can produce a more nearly uniform field by using *two* such loops a distance d apart (Fig. 5.59).
(a) Find the field (B) as a function of z, and show that $\partial B/\partial z$ is zero at the point midway between them ($z = 0$).
(b) If you pick d just right, the *second* derivative of B will *also* vanish at the mid-point. This arrangement is known as a **Helmholtz coil**; it's a convenient way of producing relatively uniform fields in the laboratory. Determine d such that $\partial^2 B/\partial z^2 = 0$ at the midpoint, and find the resulting magnetic field at the center. [*Answer*: $8\mu_0 I/5\sqrt{5}R$]

8- [Griffiths 5.50] Magnetostatics treats the "source current" (the one that sets up the field) and the "recipient current" (the one that experiences the force) so asymmetrically that it is by no means obvious that the magnetic force between two current loops is consistent with Newton's third law. Show, starting with the Biot-Savart law (Eq. 5.34) and the Lorentz force law (Eq. 5.16), that the force on loop 2 due to loop 1 (Fig. 5.61) can be written as

$$\mathbf{F}_2 = -\frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} d\mathbf{l}_1 \cdot d\mathbf{l}_1. \tag{3}$$

In this form, it is clear that $\mathbf{F}_2 = -\mathbf{F}_1$, since $\hat{\boldsymbol{\mathscr{r}}}$ changes direction when the roles of 1 and 2 are interchanged. (If you seem to be getting an "extra" term, it will help to note that $d\mathbf{l}_2 \cdot \hat{\boldsymbol{\mathscr{r}}} = d\mathscr{r}$.)

***9-** [Griffiths 5.22] Suppose there did exist magnetic monopoles. How would you modify Maxwell's equations and the force law to accommodate them? If you think there are several plausible options, list them, and suggest how you might decide experimentally which one is right.

10- [Griffiths 5.25] If $\mathbf{B}$ is *uniform*, show that $\mathbf{A}(\mathbf{r}) = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$ works. That is, check that $\nabla \cdot \mathbf{A} = 0$ and $\nabla \times \mathbf{A} = \mathbf{B}$. Is this result unique, or are there other functions with the same divergence and curl?

***11-** [Griffiths 5.29] Suppose you want to define a magnetic scalar potential U (Eq. 5.67) in the vicinity of a current-carrying wire. First of all, you must stay away from the wire itself (there $\nabla \times \mathbf{B} = 0$); but that's not enough. Show, by applying Ampère's law to a path that starts at $\mathbf{a}$ and circles the wire, returning to $\mathbf{b}$ (Fig. 5.47), that the scalar potential cannot be single-valued (that is, $U(\mathbf{a}) = U(\mathbf{b})$, even if they represent the same physical point). As an example, find the scalar potential for an infinite straight wire. (To avoid a multivalued potential, you must restrict yourself to simply-connected regions that remain on one side or the other of every wire, never allowing you to go all the way around.)

12- [Griffiths 5.33] Prove

$$\frac{\partial \mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial \mathbf{A}_{\text{below}}}{\partial n} = -\mu_0 \mathbf{K}. \tag{4}$$

using $\nabla \cdot \mathbf{A} = 0$, $\mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0(\mathbf{K} \times \hat{\mathbf{n}})$, and $\mathbf{A}_{\text{above}} = \mathbf{A}_{\text{below}}$. [Suggestion: I'd set up Cartesian coordinates at the surface, with z perpendicular to the surface and x parallel to the current.]