Problem Set 7

due Sunday 5th of Day in class

1- Consider Maxwell's equations with magnetic monopoles. Show that the continuity equation for electric charge (charge conservation) still holds. Show that the magnetic charge also satisfies the continuity equation: $\partial\rho_m/\partial t + \nabla\cdot\mathbf{J}_m = 0$.

2- Consider Maxwell's equations and Lorentz force law with magnetic monopoles. Show that magnetic fields have vanishing work on *electric* charges. Similarly show that electric fields have vanishing work on *magnetic* charges. Follow the derivation of Poynting's equation and explain the difference from the standard (no magnetic monopoles) case.

3- Recall the proof that led to the existence of scalar and vector potentials (section 6.2 in Jackson). Is it still valid in the presence of magnetic monopoles?

4- The quantization condition[1]

$$\frac{\mu_0 q_{e1} q_{m2}}{2\pi\hbar} \in \mathbb{Z} \tag{1}$$

was derived under the assumption that particle 1 has zero magnetic charge ($q_{m1} = 0$), and in general is inconsistent with the symmetry transformation

$$\begin{pmatrix} cq'_e \\ q'_m \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} cq_e \\ q_m \end{pmatrix}, \qquad \begin{pmatrix} \mathbf{E}' \\ c\mathbf{B}' \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ c\mathbf{B} \end{pmatrix}. \tag{2}$$

Now consider two particles, with electric charge q_{e1} and q_{e2}, as well as magnetic charges q_{m1} and q_{m2}. Repeat the analysis of Section 6.12 of Jackson to obtain the change in the angular momentum of particle 1 (when particle 2 is held fixed) and show that

$$\frac{\mu_0(q_{e1}q_{m2} - q_{e2}q_{m1})}{2\pi\hbar} \in \mathbb{Z}. \tag{3}$$

Check that this is consistent with the symmetry transformation above.

[1] This is in the Ampere-meter convention, where magnetic charge has units of Am. There is no μ_0 in the corresponding formula in the Weber convention (employed in Jackson), where magnetic charge has units of magnetic flux (Wb).