Electrodynamics 1

Problem Set 7

due Saturday 30th of Azar in class

1- Using the definitions

$$U^\mu = \frac{dx^\mu}{d\tau}, \qquad P^\mu = mU^\mu, \qquad F^\mu = \frac{dP^\mu}{d\tau}, \tag{1}$$

for 4-velocity, 4-momentum, and 4-force of a particle, show that

$$U^\mu = \gamma(u)(c, \mathbf{u}), \qquad P^\mu = (c^{-1}E, \mathbf{p}), \qquad F^\mu = \gamma(c^{-1}dE/dt, \mathbf{f}). \tag{2}$$

Here $\mathbf{u} = d\mathbf{x}/dt$ is the 3-velocity, $E = \gamma mc^2$ is the relativistic energy, $\mathbf{p} = \gamma m\mathbf{u}$ is the (relativistic) 3-momentum, and $\mathbf{f} = d\mathbf{p}/dt$ is the 3-force. [Hint: Prove that $d\tau = dt/\gamma(u)$.]

2- Compute the invariants $U_\mu U^\mu$ and $P_\mu P^\mu$.

3- Show that the rest mass m of the particle doesn't change in time if and only if $F_\mu U^\mu = 0$. Prove that the Lorentz force $F_\mu = \frac{q}{c}F_{\mu\nu}U^\nu$ is rest-mass preserving.

4- [Jackson 11.14] a) Express the Lorentz scalars $F^{\alpha\beta}F_{\alpha\beta}$, $\mathcal{F}^{\alpha\beta}F_{\alpha\beta}$ and $\mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta}$ in terms of $\mathbf{E}$ and $\mathbf{B}$. Are there any other invariants quadratic in the field strengths $\mathbf{E}$ and $\mathbf{B}$?

b) Is it possible to have an electromagnetic field that appears as a purely electric field in one inertial frame and as a purely magnetic field in some other inertial frame? What are the criteria imposed on $\mathbf{E}$ and $\mathbf{B}$ such that there is an inertial frame in which there is no electric field?

[. . .]

5- Using Lorentz transformations find the force between two electric charges q which move side by side along parallel lines with velocity v.