Electrodynamics 1

Problem Set 4

due Saturday 25th of Aban in class

1- Consider three charges on the z-axis: $+q$ at $z = a$, $-2q$ at $z = 0$, and $+q$ at $z = -a$ (see figure (b) of Problem 4.1 in Jackson). Compute all multipole moments of this charge configuration with $l = 0, 1, 2$. Now take the limit $a \to 0$ but keep qa^2 constant. Show that all moments with $l > 2$ vanish in this limit.

2- [Jackson 4.5] A localized charge density $\rho(x, y, z)$ is placed in an external electrostatic field described by a potential $\Phi^{(0)}(x, y, z)$. The external potential varies slowly in space over the region where the charge density is different from zero.

a) From first principles calculate the total *force* acting on the charge distribution as an expansion in multipole moments times derivatives of the electric field, up to and including the quadrupole moments. Show that the force is

$$\mathbf{F} = q\mathbf{E}^{(0)}(0) + \{\nabla[\mathbf{p} \cdot \mathbf{E}^{(0)}(\mathbf{x})]\}_0 + \left\{\nabla \left[\frac{1}{6}\sum_{jk} Q_{jk} \frac{\partial E_j^{(0)}}{\partial x_k}(\mathbf{x})\right]\right\}_0 + \cdots \tag{1}$$

Compare this to the expansion (4.24) of the energy W. Note that (4.24) is a number—it is not a function of $\mathbf{x}$ that can be differentiated! What is its connection to $\mathbf{F}$?

b) Repeat the calculation of part (a) for the total torque. For simplicity, evaluate only one Cartesian component of the torque, say N_1. Show that this component is

$$N_1 = [\mathbf{p} \cdot \mathbf{E}^{(0)}(0)]_1 + \frac{1}{3}\left[\frac{\partial}{\partial x_3}\left(\sum_j Q_{2j} E_j^{(0)}\right) - \frac{\partial}{\partial x_2}\left(\sum_j Q_{3j} E_j^{(0)}\right)\right]_0 + \cdots \tag{2}$$

3- [Jackson 4.9] A point charge q is located in free space a distance d from the center of a dielectric sphere of radius a ($a < d$) and dielectric constant ϵ/ϵ_0.

a) Find the potential at all points in space as an expansion in spherical harmonics.

b) Calculate the rectangular components of the electric field *near* the center of the sphere.

c) Verify that, in the limit $\epsilon/\epsilon_0 \to \infty$, your result is the same as that for the conducting sphere.

4- [Jackson 4.12] Water vapor is a polar gas whose dielectric constant exhibits an appreciable temperature dependence. The following table gives experimental data on this effect. Assuming that water vapor obeys the ideal gas law, calculate the molecular polarizability as a function of inverse temperature and plot it. From the slope of the curve, deduce a value for the permanent dipole moment of the H_2O molecule (express the dipole moment in coulomb-meters).

T(K)	Pressure (cm Hg)	$(\epsilon/\epsilon_0 - 1) \times 10^5$
393	56.49	400.2
423	60.93	371.7
453	65.34	348.8
483	69.75	328.7