Electrodynamics 1

Problem Set 2

due Monday 22nd of Mehr in class

1- Obtain the Green function for the Dirichlet problem inside of a sphere. [See section 2.6 of Jackson.]

2- [Similar to Jackson 2.1 and 2.7a] a) Apply the method of images to a point charge q at $\mathbf{x}'$ near an infinite plane conductor held at zero potential. Find the potential at point $\mathbf{x}$ on the same side of the plane as q.

b) Using the result of part (a) find the Green function for the Dirichlet problem on one side of a plane.

3- Consider two conducting planes, one at $x = 0$ and the other at $y = 0$. A line of charge with uniform density λ is placed on the bisector of the first quadrant at $x = y = d > 0$.

a) Find image charges to obtain the potential in the first quadrant $x, y > 0$. [Hint: See the figure of problem 2.20 in Jackson.]

b) Approximate your answer in part (a) near the corner $x = y = 0$. Does this match the general result of section 2.11?

c) Can you use the result of part (a) to obtain the Green function for the Dirichlet problem in the first quadrant? No explicit calculation is required; just explain.

4- In the class I outlined how to find the potential for a ring of charge (Jackson figure 3.4). Fill in the gaps and complete the derivation.

5- [optional, not to be graded; Jackson 3.5] A hollow sphere of inner radius a has the potential specified on its surface to be $\Phi = V(\theta, \phi)$. Prove the equivalence of the two forms of solution for the potential inside the sphere:

a)

$$\Phi(\mathbf{x}) = \frac{a(a^2 - r^2)}{4\pi} \int \frac{V(\theta', \phi')}{(r^2 + a^2 - 2ar\cos\gamma)^{3/2}} d\Omega' \tag{1}$$

where $\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi')$.

b)

$$\Phi(\mathbf{x}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm} \left(\frac{r}{a}\right)^l Y_{lm}(\theta, \phi) \tag{2}$$

where $A_{lm} = \int d\Omega' Y^*_{lm}(\theta', \phi') V(\theta', \phi')$.