فیزیک قطعات نیمه رسانا

HW #1

حل تمارین سری اول ، ترم پاییز ۹۵-۹۶

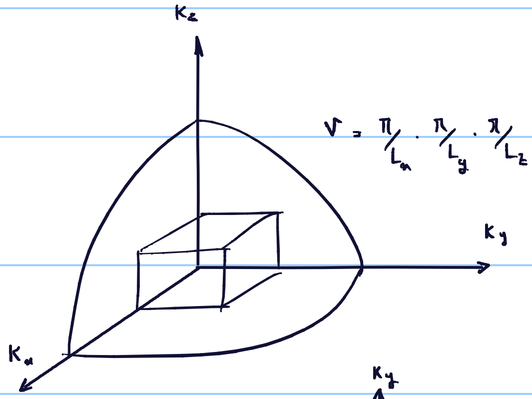
Note Title

2. $V(x,y,z) = 0$ for $0 < x < L$

$0 < y < L$

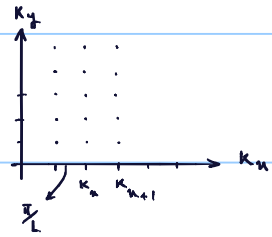
$0 < z < L$

$V(x,y,z) = \infty$ elsewhere



$V = \pi/L_x \cdot \pi/L_y \cdot \pi/L_z$

$$\frac{2mE}{\hbar^2} = k^2 = k_x^2 + k_y^2 + k_z^2 = (n_x^2 + n_y^2 + n_z^2)\left(\frac{\pi^2}{L^2}\right)$$



فاصله‌ی هر دو حالت کوانتومی در فضای K

$$* \; k_{n+1} - k_n = (n_x + 1)\left(\pi/L\right) - n_x\left(\pi/L\right) = \pi/L \quad \Rightarrow \text{for 3D: } V_k = \left(\pi/L\right)^3$$

$$N(k)\,dk = 2\left(\frac{1}{8}\right)\left(\frac{4\pi k^2 dk}{(\pi/L)^3}\right) \Rightarrow N(k)\,dk = \frac{\pi k^2 dk}{\pi^3} \cdot L^3$$

$$k^2 = \frac{2mE}{\hbar^2} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \frac{1}{\hbar}\sqrt{\frac{m}{2E}}\, dE$$

$$\Rightarrow N(E)\,dE = \frac{\pi L^3}{\pi^3} \cdot \left(\frac{2mE}{\hbar^2}\right)^{3/2} \frac{1}{\hbar}\sqrt{\frac{m}{2E}}\, dE$$

حالت‌های کوانتومی بین انرژی E, $E+dE$

$$\hbar = \frac{h}{2\pi} \Rightarrow N(E)\,dE = \frac{4\pi L^3}{h^3} \cdot (2m)^{3/2} \cdot \sqrt{E}\, dE \Rightarrow N(E) = \frac{8\pi L^3}{3h^3}(2mE)^{3/2}$$

$$Dos_{3D} = \frac{dN}{dE} \cdot \frac{1}{V} \Rightarrow Dos = \frac{4\pi (2m)^{3/2}}{h^3} \cdot \sqrt{E}$$

b. $E = \dfrac{\hbar^2 k^2}{2m}$ @ $k = 0 \Rightarrow E = E_c$

نوار رسانش : $E = E_c + \dfrac{\hbar^2 k^2}{2m_n^*} \Rightarrow E - E_c = \dfrac{\hbar^2 k^2}{2m_n^*}$

$$\Rightarrow Dos_{3D-c} = \frac{4\pi (2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

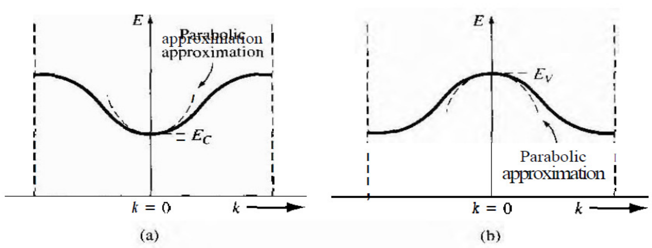


Figure 3.16 | (a) The conduction band in reduced k space, and the parabolic approximation. (b) The valence band in reduced k space, and the parabolic approximation.

نوار ظرفیت : $E = E_c - \dfrac{\hbar^2 k^2}{2m_n^*} \Rightarrow E_v - E = \dfrac{\hbar^2 k^2}{2m_p^*}$

$$\Rightarrow Dos_{3D-v} = \frac{4\pi (2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E_c}$$

3. $E_1 = E_f + \Delta E$

$$f_1(E_1) = \frac{1}{1+\exp\left(\frac{E_1-E_f}{kT}\right)} = \frac{1}{1+\exp\left(\frac{\Delta E}{kT}\right)} \quad , \quad E_2 = E_f - \Delta E$$

$$1 - f_2(E_2) = 1 - \frac{1}{1+\exp\left(\frac{E_2-E_f}{kT}\right)} = 1 - \frac{1}{1+\exp\left(\frac{-\Delta E}{kT}\right)} = \frac{\exp\left(\frac{-\Delta E}{kT}\right)}{1+\exp\left(\frac{-\Delta E}{kT}\right)}$$

$$\Rightarrow 1 - f_2(E_2) = \frac{1}{1+\exp\left(\frac{\Delta E}{kT}\right)} \Rightarrow f_1(E_1) = 1 - f_2(E_2) \quad \text{Q.E.D.}$$

4. $D_e(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}(\varepsilon - E_c)^{1/2} \quad , \quad f(\varepsilon) = e^{(\mu-\varepsilon)}$

$$n = \int_{E_c}^{\infty} D(\varepsilon) f(\varepsilon)\, d\varepsilon = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} e^{\mu/k_BT} \int_{E_c}^{\infty} (\varepsilon - E_c)^{1/2} e^{-\varepsilon/k_BT} d\varepsilon$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} e^{\mu/k_BT} \underbrace{\int_{E_c}^{\infty} (\varepsilon - E_c)^{1/2} e^{-\varepsilon/k_BT} d\varepsilon}_{I} \Rightarrow n = \frac{1}{\sqrt{2}}\left(\frac{m_e^* k_BT}{\pi\hbar^2}\right)^{3/2} e^{(\mu - E_c)/k_BT}$$

$$* \; I = \int_{0}^{\infty} x^{1/2} e^{-(x+E_c)/k_BT} dx = e^{-E_c/k_BT} \underbrace{\int_0^{\infty} x^{1/2} e^{-x/k_BT} dx}_{(k_BT)^{3/2}\frac{1}{2}\Gamma(\frac{1}{2})} = \frac{\sqrt{\pi}}{2} e^{-E_c/k_BT}$$

$\varepsilon - E_c = x$

$$D_h(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}(E_v - \varepsilon)^{1/2}$$

$$\Rightarrow P = \int_{-\infty}^{E_v} D_h(\varepsilon) f_h(\varepsilon)\, d\varepsilon = 2\left(\frac{m_h^* k_BT}{2\pi\hbar^2}\right)^{3/2} e^{(E_c-\mu)/k_BT}$$

$$\Rightarrow n.p = 4\left(\frac{k_BT}{2\pi\hbar^2}\right)^{3}(m_e^* m_h^*)^{3/2} e^{-E_g/k_BT}$$

$n = p$ ذاتی $\Rightarrow n_i = p_i = 2\left(\frac{k_BT}{2\pi\hbar^2}\right)^{3/2}(m_e^* m_h^*)^{3/4} e^{-E_g/2k_BT}$

5. $\frac{m}{\tau} v_x + \frac{eB}{c} v_y = -eE_x$; $\frac{m}{\tau} v_y - \frac{eB}{c} v_x = -eE_y$; $\frac{m}{\tau} v_z = -eE_z$

$$\begin{pmatrix} 1 & \omega_c\tau & 0 \\ -\omega_c\tau & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{-e\tau}{m}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau_e & 0 \\ \omega_c\tau_e & 1 & 0 \\ 0 & 0 & (1+(\omega_c\tau)^2)^{-1} \end{pmatrix}$$

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \sigma_e\begin{pmatrix} 1 & -\omega_c\tau_e & 0 \\ \omega_c\tau_e & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$(\omega_c\tau)^2 \simeq 0$ and $B^2 \simeq 0$

for holes: $e \to -e$; $m_e \to m_h$; $\tau_e \to \tau_h$

$w_c^* = \frac{eB}{m_h}$, $\sigma_h = ne^2\tau_h/m_h$

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \sigma_e\begin{pmatrix} 1 & \omega_c\tau_h & 0 \\ -\omega_c\tau_h & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$j_z = 0$ if $E_z = 0$

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_e+\sigma_h & \sigma_h w_c^*\tau_h - \sigma_e w_c\tau_e \\ -\sigma_h w_c^*\tau_h + \sigma_e w_c\tau_e & \sigma_e+\sigma_h \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$\mu_e = \frac{e\tau_e}{m_e}$, $\mu_h = \frac{e\tau_h}{m_h}$

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = e\begin{pmatrix} (n\mu_e+p\mu_h) & -B(n\mu_e^2-p\mu_h^2) \\ B(n\mu_e^2-p\mu_h^2) & (n\mu_e+p\mu_h) \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

@ Equilibrium $j_y = 0 \Rightarrow E_x = -\frac{n\mu_e+p\mu_h}{B(n\mu_e^2-p\mu_h^2)} E_y$

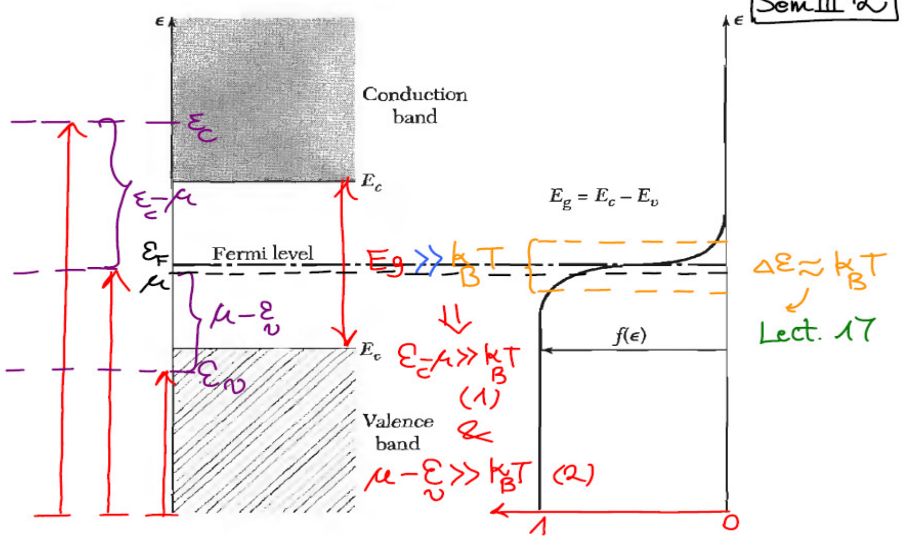
$j_x = -\frac{e}{B}\frac{(n\mu_e+p\mu_h)^2}{(n\mu_e^2-p\mu_h^2)} + O(B) \Rightarrow R_H = \frac{E_y}{j_x B} = -\frac{n\mu_e^2-p\mu_h^2}{e(n\mu_e+p\mu_h)^2}$ if $b = \mu_e/\mu_h \Rightarrow R_H = \frac{p-nb^2}{e(nb+p)^2}$

پیوست :

$$f_e = \frac{1}{\exp[(\varepsilon-\mu)/k_BT]+1} \Rightarrow f_e = \frac{e^{(\mu-\varepsilon)/k_BT}}{e^{(\mu-\varepsilon)/k_BT}} \times \frac{1}{\exp[(\varepsilon-\mu)/k_BT]+1} = e^{(\mu-\varepsilon)/k_BT}\,\frac{1}{1+\underbrace{e^{(\mu-\varepsilon)/k_BT}}_{\to 0}}$$

$$\Rightarrow f_e \simeq e^{(\mu-\varepsilon)/k_BT} \qquad \text{if } \frac{\mu-\varepsilon_c}{k_BT} \to \infty$$

Sem III 2



$$\lim_{T\to 0} \mu(T) = \varepsilon_f$$

So @ $T \neq 0$

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